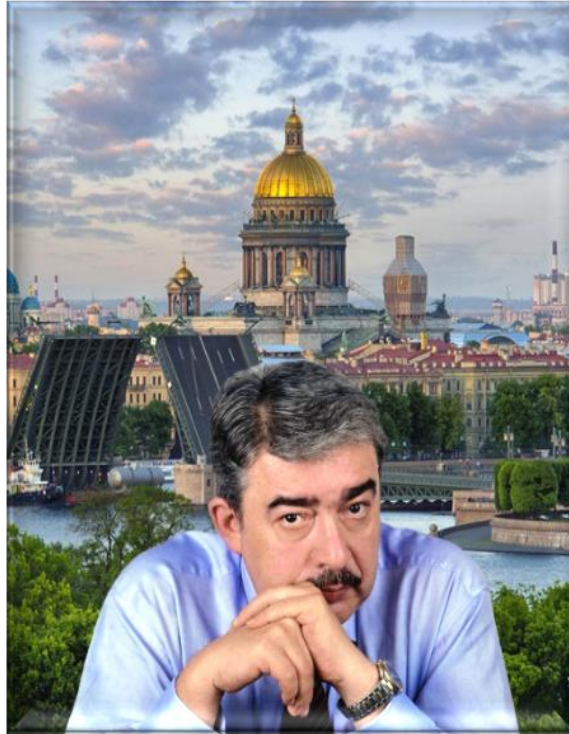


## The Golden Ratio and Symmetry

*(Compilation from article "Golden Section")*

Author: Shkrudnev F.D.



Fedor Dmitrievich Shkrudnev

In his works, Fedor Dmitrievich draws attention to the phenomenon of the golden section, which has been undeservedly forgotten by modern science. However, the source of new scientific ideas and discoveries is still hidden in the study of natural objects that develop according to the laws of the golden section and symmetry.

\* \* \*

The golden Ratio cannot be considered by itself, separately, without connection with symmetry.

The great Russian crystallographer G. V. Wulf (1863-1925) considered the golden section to be one of the manifestations of symmetry.

The golden division is not a manifestation of asymmetry, something opposite to symmetry. According to modern concepts, the golden division is an asymmetric symmetry. The science of symmetry includes such concepts as static and dynamic symmetry. Static symmetry characterizes rest, balance, and dynamic-movement, growth. So, in nature, static symmetry is represented by the structure of crystals, and in art it characterizes peace, balance and immobility. Dynamic symmetry expresses activity, characterizes movement, development, rhythm, it is a testimony of life. Static symmetry is characterized by equal segments, equal values. Dynamic symmetry is characterized by an increase in segments or their decrease, and it is expressed in the values of the golden section of an increasing or decreasing series.

### Fibonacci Series

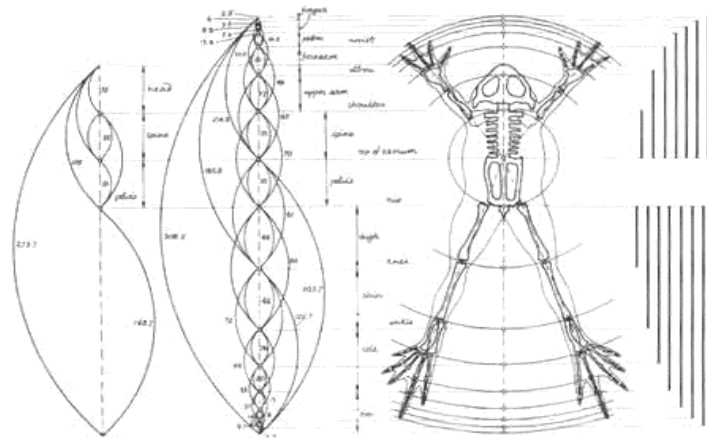
The name of the Italian mathematician monk Leonardo from Pisa, better known as Fibonacci, is indirectly connected with the history of the golden section. He traveled a lot in the East and introduced Europe to Arabic numerals. In 1202, his mathematical work "The Book of the Abacus" (counting board) was published, in which all the problems known at that time were collected.

A series of numbers 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55 etc. is known as the Fibonacci series.

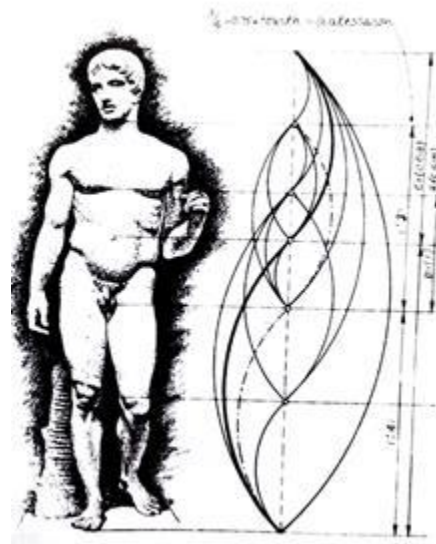
The peculiarity of the sequence of numbers is that each of its members, starting from the third, is equal to the sum of the previous two  $2 + 3 = 5$ ;  $3 + 5 = 8$ ;  $5 + 8 = 13$ ;  $8 + 13 = 21$ ;  $13 + 21 = 34$  and so on, and the ratio of adjacent numbers of the series approaches the ratio of the golden division. So,  $21 \div 34 = 0.617$ , and  $34 \div 55 = 0.618$ . This relation is denoted by the symbol  $F$ . Only this ratio -  $0.618 \div 0.382$  - gives a continuous division of a straight line segment in the golden ratio, increasing it or decreasing it with no finitude, when the smaller segment relates to the larger one as the larger one relates to everything.

For example, the length of each finger joint corresponds to the length of the next joint in the proportion of  $F$ . The same ratio is shown in all fingers and toes. This connection is somehow unusual, because one finger is longer than the other without any apparent regularity, but this is not accidental, just as everything in the human body is not accidental. The distances on the fingers marked from A to B to C to D to E all correspond to each other in the proportion of  $F$ , as well as the phalanges of the fingers from F to G to H.

Take a look at this frog skeleton (Fig. 1.) and see how each bone corresponds to the model of the proportion  $F$  exactly as in the human body (Fig. 2.).



*Fig. 1. Proportion of F in the skeleton of a frog*



*Fig. 2. The proportion of F in the parts of the human body*

### **Principles of formation in nature**

Everything that acquired some shape, formed, grew and sought to take a place in space and preserve itself. This aspiration is realized mainly in two ways: growing up or spreading out on the surface of the earth and twisting in a spiral.

A shell is twisted in a spiral. If you expand it, you get a length that is slightly inferior to the length of a snake. A small ten-centimeter shell has a spiral length of 35 cm. Spirals (Fig. 3.) are very common in nature. The idea of the golden section will be incomplete, if we did not mention the spiral.

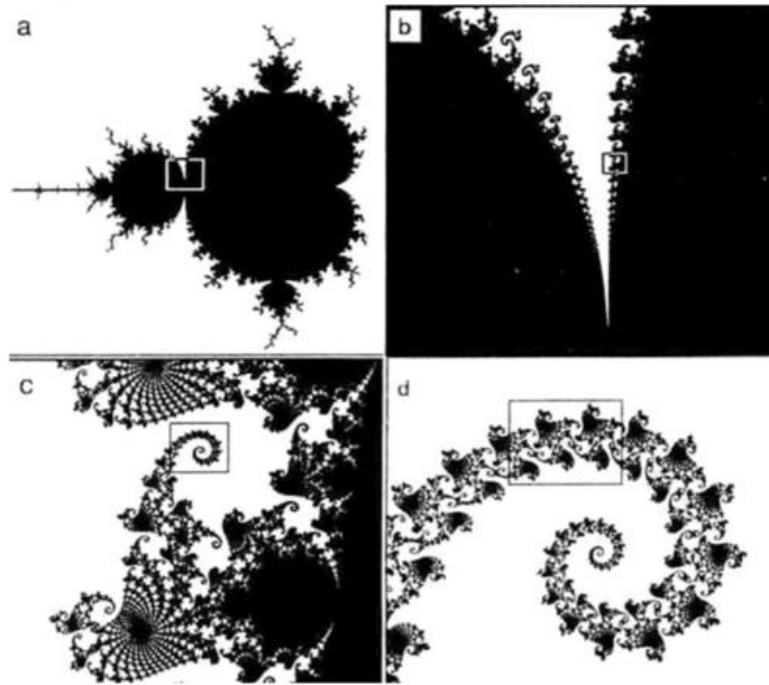
The shape of the spirally curled shell attracted the attention of Archimedes. He studied it and derived the equation of the spiral. The spiral drawn by this equation is named after him. The increase in its step is always uniform. Currently, the Archimedes spiral is widely used in engineering. Goethe also emphasized the tendency of nature to spiral, calling the spiral the "curve of life". The helical and spiral arrangement of leaves on tree branches has been observed for a long time.



*Fig. 3. The spiral is one of the most common forms in nature*

The spiral was seen in the arrangement of sunflower seeds, pine cones, pineapples, cacti, etc. The joint work of botanists and mathematicians shed light on these amazing natural phenomena. It turned out that the Fibonacci series manifests itself in the arrangement of leaves on a branch (phyllotaxis), sunflower seeds, and pine cones and therefore, the law of the golden section manifests itself. The spider weaves a web in a spiral manner. A hurricane is spiraling. A frightened herd of reindeer runs away in a spiral. The DNA molecule is twisted in a double helix.

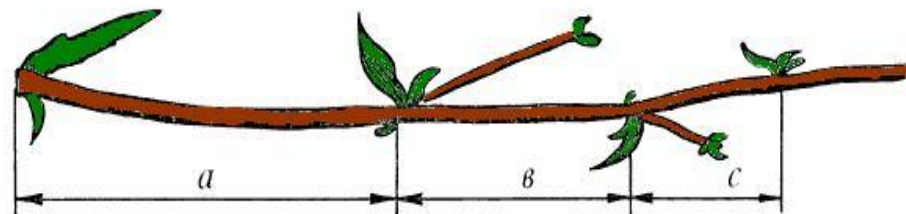
The golden spiral is closely related to cycles. The modern science of chaos studies simple cyclic feedback operations and the fractal forms generated by them, which were previously unknown.



*Fig. 4. Mandelbrot fractals*

The figure shows the well-known Mandelbrot series (Fig. 4.) - a page from the dictionary without finitude of individual patterns called the Julian series. Some scientists associate the Mandelbrot formula with the genetic code of cell nuclei. The successive increase in the cross-sections reveals fractals that are amazing in their artistic complexity. And here, too, there are logarithmic spirals! This is all the more important because both the Mandelbrot series and the Julian series are not an invention of the human mind. They arise from the domain of Plato's primordial images. As R. Penrose said, "they are like Mount Everest".

Among the roadside grasses, a seemingly unremarkable plant grows - chicory. Let's look at it carefully. A process was formed from the main stem (Fig. 5.). The first leaf was located right there.



*Fig. 5. Proportions of the "golden section" in Plants*

The process makes a strong ejection into space, stops, releases a leaf, but shorter than the first one, again makes an ejection into space, but of a lesser force, releases a leaf of an even smaller size and again an ejection.

If the first ejection is taken as 100 units, then the second is equal to 62 units, the third is 38, the fourth is 24, etc. The length of the petals is also subject to the golden proportion. In the growth, the conquest of space, the plant maintained certain proportions. The impulses of its growth gradually decreased in proportion to the golden ratio.

In many butterflies, the ratio of the size of the thoracic and abdominal parts of the body corresponds to the golden proportion (Fig. 6.). Folding the wings, the moth forms a regular equilateral triangle. But it is worth spreading the wings, and you will see the same principle of dividing the body into 2, 3, 5, 8. The dragonfly is also created according to the laws of the golden proportion: the ratio of the lengths of the tail and the body is equal to the ratio of the total length to the length of the tail.

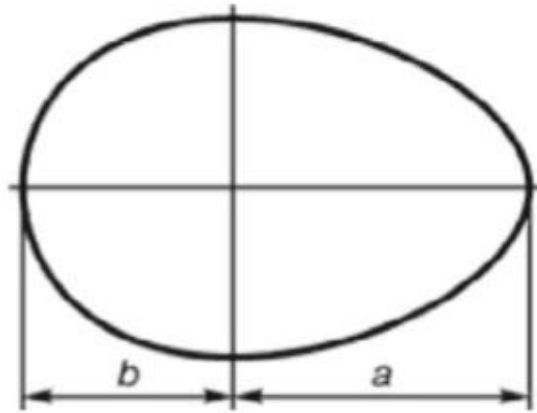


*Fig. 6. The regularities of the "golden" proportion are manifested in the proportions of insects*

At first glance, the proportions that are pleasant to our eyes are captured in the lizard - the length of its tail is as related to the length of the rest of the body, 62 to 38.

In other words, both in the plant and animal world, the formative tendency of nature - symmetry with respect to the direction of growth and movement - persistently breaks through. Here, the golden ratio is manifested in the proportions of the parts perpendicular to the direction of growth.

Nature has carried out the division into symmetrical parts and golden proportions. In the parts, the repetition of the structure of the whole is manifested. Of great interest is the study of the shapes of bird eggs (Fig. 7.). Their various shapes fluctuate between two extreme types: one of them can be inscribed in a rectangle of the golden section, the other in a rectangle with a modulus of 1.272 (the square root of the golden proportion).



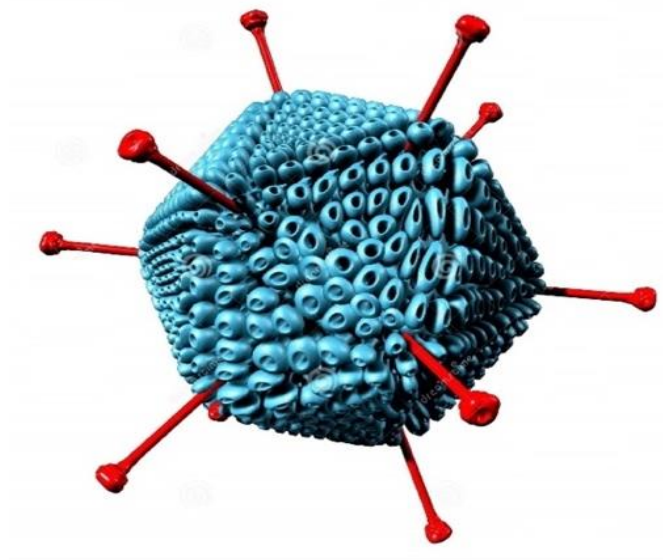
*Fig. 7. The "Golden section" is also present in the form of an egg*

Such shapes of bird eggs are not accidental, since it has now been established that the shape of eggs described by the golden ratio corresponds to higher strength characteristics of the egg shell.

The tusks of elephants and extinct mammoths, the claws of lions, and the beaks of parrots are logarithmic shapes and resemble the shape of an axis that tends to turn into a spiral. In living nature, forms based on "pentagonal" symmetry (starfish, sea urchins, flowers) are widely distributed.

The golden ratio is present in the structure of all crystals, but most crystals are microscopically small, so we cannot see them with the naked eye. However, snowflakes, which are also water crystals, are quite accessible to our eyes. All the exquisite beauty of the figures that form snowflakes, all the axes, circles and geometric shapes in snowflakes are also always, without exception, built according to the perfect clear formula of the golden section.

In the microcosm, three-dimensional logarithmic forms constructed according to golden proportions are widespread everywhere. For example, many viruses have a three-dimensional geometric shape of an icosahedron. Perhaps the most famous of these viruses is the Adeno virus (Fig. 8.). The protein envelope of the Adeno virus is formed from 252 units of protein cells arranged in a certain sequence. In each corner of the icosahedron, there are 12 units of protein cells in the form of a pentagonal prism, and spiked structures extend from these corners.

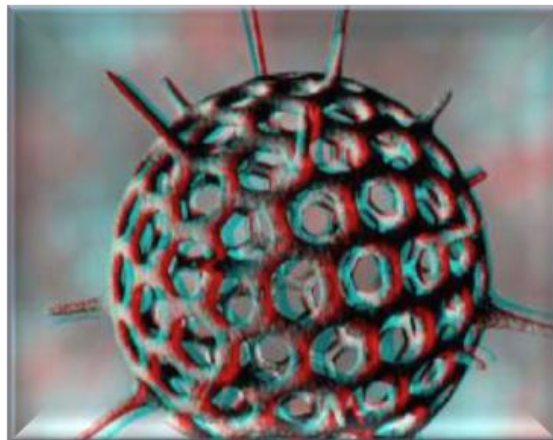


*Fig. 8. Adeno virus*

For the first time in the 1950s, the golden section was discovered in the structure of viruses by scientists A. Klug and D. Kaspar from Birkbeck College in London. The first logarithmic form was revealed by the Polyo virus. The form of this virus turned out to be similar to the form of the Rhino virus.

The question arises: how do viruses form such complex three-dimensional forms, the device of which contains the golden section, which even in our human mind is quite difficult to construct? The discoverer of these forms of viruses, virologist A. Klug, makes the following comment: "Dr. Kaspar and I have shown that for a spherical shell of a virus, the most optimal form is the symmetry of the icosahedron type shape. This order minimizes the number of connecting elements... Most of the Buckminster Fuller geodesic hemispherical cubes are constructed according to a similar geometric principle. The installation of such cubes requires an extremely accurate and detailed explanation scheme, whereas unconscious viruses themselves construct such a complex shell of elastic, flexible protein cellular units".





*Fig. 9. The complex fullerene structure of protein molecules*

A. Kluge's comment once again reminds us of the extremely obvious truth: in the structure of even a microscopic organism, which scientists classify as the "most primitive form of life", in this case in a virus, there is a clear plan and a reasonable project has been implemented. This project is incomparable in its perfection and accuracy of execution with the most advanced architectural projects created by people. For example, projects created by the brilliant architect Bookminster Fuller.

Three-dimensional models of the dodecahedron and icosahedron are also present in the structure of the skeletons of single-celled marine microorganisms of radiolarians (radiophores), the skeleton of which is made of silica.

Radiolarians form their body of a very refined, unusual beauty. Their shape is a regular dodecahedron, and from each corner of it sprouts a pseudo-elongation-a limb and other unusual forms-growths.

The great Goethe, a poet, naturalist and artist (he drew and painted in watercolors), dreamed of creating a unified doctrine about the form, formation and transformation of organic bodies. It was he who introduced the term morphology into scientific use. At the beginning of this century, Pierre Curie formulated a number of profound ideas of symmetry. He argued that it is impossible to consider the symmetry of any body without taking into account the symmetry of the environment.

The regularities of the "golden" symmetry are manifested in the energy transitions of elementary particles, in the structure of some chemical compounds, in planetary and cosmic systems, in the gene structures of living organisms. These patterns, as indicated above, exist in the structure of individual human organs and the body as a whole, and also manifest themselves in the biorhythms and functioning of the brain and visual perception.

## The human body and the golden ratio

All human bones are in the proportion of the golden ratio. The proportions of the various parts of our body make up a number very close to the golden ratio. If these proportions coincide with the formula of the golden ratio, then the appearance or body of a person is considered perfectly folded (Fig. 10.).

If we take the navel point as the center of the human body, and the distance between a person's foot and the navel point as a unit of measurement, then the height of a person is equivalent to the number 1,618:

the distance from the shoulder level to the top of the head and the size of the head is  $1 \div 1,618$ ;

the distance from the navel point to the top of the head and from the shoulder level to the top of the head is  $1 \div 1,618$ ;

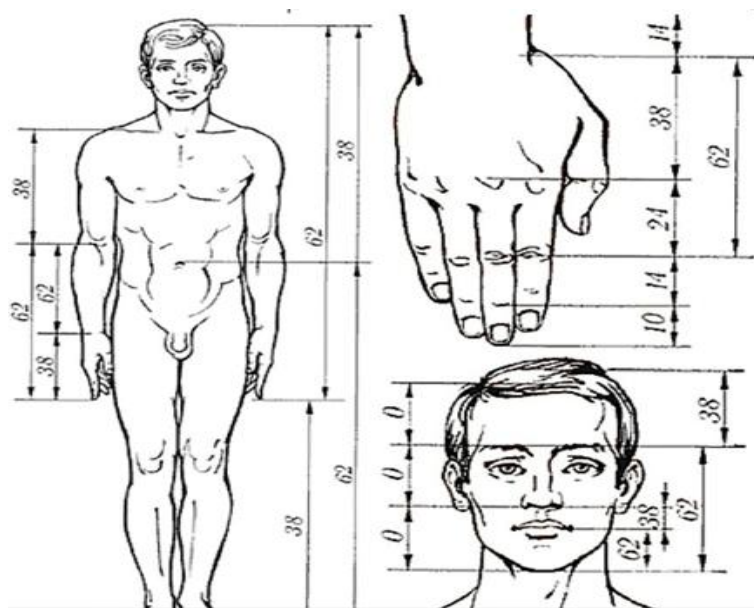
the distance of the navel point to the knees and from the knees to the feet is  $1 \div 1,618$ ;

the distance from the tip of the chin to the tip of the upper lip and from the tip of the upper lip to the nostrils is  $1 \div 1,618$ ;

actually, the exact presence of a golden proportion in a person's face is the ideal of beauty for the human eye;

the distance from the tip of the chin to the upper line of the eyebrows and from the upper line of the eyebrows to the top of the head is  $1 \div 1,618$ ;

the distance from the tip of the chin to the upper line of the eyebrows and from the upper line of the eyebrows to the top of the head is  $1 \div 1,618$ .



*Fig. 10. The human body and the golden ratio*

Face height/face width:

**the central point of the junction of the lips to the base of the nose/the length of the nose;**  
**face height/distance from the tip of the chin to the central point of the junction of the lips;**  
**mouth width/nose width;**  
**nose width/distance between nostrils;**  
**the distance between the pupils/the distance between the eyebrows.**

It is enough just to bring your palm closer to yourself now and look carefully at the index finger, and you will immediately find the formula of the golden ratio in it.

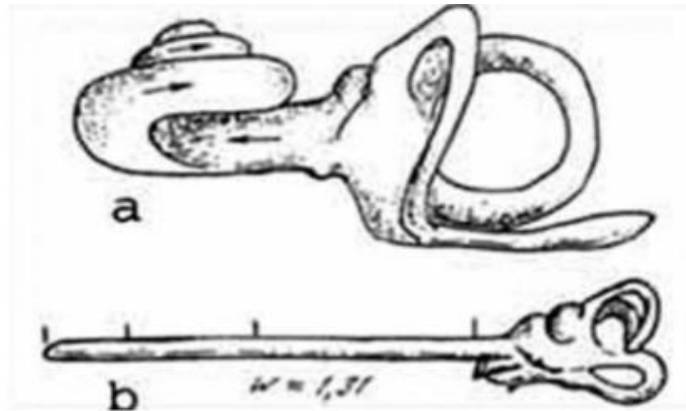
Each finger of our hand consists of three phalanges. The sum of the length of the first two phalanges of the finger in relation to the entire length of the finger and gives the number of the golden section (except for the thumb). In addition, the ratio between the middle finger and the little finger is also equal to the number of the golden ratio.

A person has two hands; the fingers on each hand consist of three phalanges (with the exception of the thumb). There are five fingers on each hand, that is, only 10, but with the exception of the two phalangeal thumbs, only eight fingers are created according to the principle of the golden section. Whereas all these numbers 2, 3, 5 and 8 are numbers of the Fibonacci sequence.

It should also be noted that for most people, the distance between the ends of their spread arms is equal to their height.

The truths of the golden section are within us and in our space. The peculiarity of the bronchi that make up the human lungs is their asymmetry. The bronchi consist of two main airways, one of which (left) is longer, and the other (right) is shorter. It was found that this asymmetry continues in the branches of the bronchi, in all the smaller airways. Moreover, the ratio of the length of short and long bronchi is also the golden ratio and is equal to  $1 \div 1,618$ .

In the inner ear of a person there is the Cochlea organ ("Snail") (Fig. 11.), which performs the function of transmitting sound vibration. This bone-like structure is filled with liquid and is also created in the form of a snail, containing a stable logarithmic spiral shape  $=73^{\circ}43'$ .



*Fig. 11. Cochlea organ  
in the human inner ear*

Blood pressure changes as the heart works. It reaches its highest value in the left ventricle of the heart at the moment of its contraction (systole). In the arteries during ventricular systole, the blood pressure reaches a maximum value of 115-125 mm Hg in a young, healthy person.

At the moment of cardiac muscle relaxation (diastole) the pressure decreases to 70-80 mmHg. The ratio of maximal (systolic) to minimal (diastolic) pressure is on average 1.6, i.e. close to the golden ratio.

If we take the average blood pressure in the aorta as a unit, then the systolic blood pressure in the aorta is 0.382, and the diastolic pressure is 0.618, i.e. their ratio corresponds to the golden proportion. This means that the work of the heart with respect to time cycles and changes in blood pressure are optimized according to the same principle of the golden ratio law.

The DNA molecule consists of two vertically intertwined spirals. The length of each of these spirals is 34 angstroms and the width is 21 angstroms. (1 angstrom is one hundred millionth of a centimeter). So, 21 and 34 are numbers following each other in the sequence of Fibonacci numbers, i.e. the ratio of length and width of the logarithmic spiral of a DNA molecule carries in itself the formula of the golden ratio  $1 \div 1,618$ .



*Fig. 12. Schematic section of the DNA helix*

### **Optimal physical parameters of the external environment**

It is known that the maximum sound volume that causes pain is 130 decibels. If we divide this interval by the golden ratio of 1.618, we get 80 decibels, which are characteristic of the loudness of a human scream. If now 80 decibels are divided by the golden ratio, we get 50 decibels, which corresponds to the volume of human speech. Finally, if we divide 50 decibels by the square of the golden ratio of 2.618, we get 20 decibels, which corresponds to a human whisper. Thus, all the characteristic parameters of sound volume are interconnected through the golden ratio.

At a temperature of 18-200°C, the humidity range of 40-60% is considered optimal. The boundaries of the optimal humidity range can be obtained if the absolute humidity of 100% is divided twice by the golden ratio:  $100/2,618 = 38.2\%$  (lower limit);  $100/1,618 = 61.8\%$  (upper limit).

At an air pressure of 0.5 MPa, a person experiences unpleasant sensations, his physical and psychological activity worsens. At a pressure of 0.3-0.35 MPa, only short-term work is allowed, and at a pressure of 0.2 MPa, he is allowed to work no more than 8 minutes. All these characteristic parameters are related by the golden ratio:  $0.5/1.618 = 0.31$  MPa;  $0.5/2.618 = 0.19$  MPa.

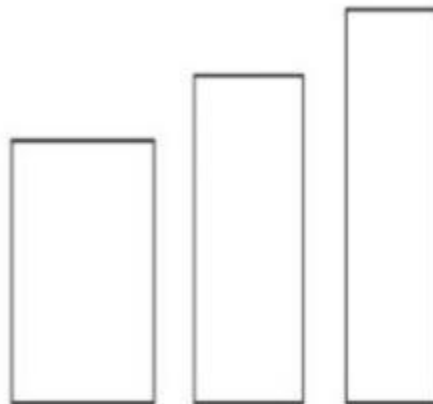
The temperature range from 0 to +(57-58)°C is the boundary parameters of the outdoor air temperature, within which the normal existence (and, most importantly, the origin) of a person. Obviously, there are no explanations for the first boundary.

We divide the specified range of positive temperatures by the golden section. In this case, we will get two boundaries (both boundaries are temperatures characteristic of the human body): the first one corresponds to the temperature; the second border corresponds to the maximum possible outdoor air temperature for the human body.

### **The Golden Ratio and image perception**

The ability of the human visual analyzer to distinguish objects constructed according to the golden section algorithm as beautiful, attractive and harmonious has been known for a long time. The golden ratio gives the feeling of the most perfect single whole. The format of many books corresponds to the golden ratio. It is chosen for windows, paintings and envelopes, stamps, business cards. A person may not know anything about the number  $F$ , but in the structure of objects, as well as in the sequence of events, he subconsciously finds elements of the golden proportion.

Studies were conducted in which the subjects were asked to select and copy rectangles of various proportions. There were three rectangles to choose from: a square ( $40 \div 40$  mm), a rectangle of the "golden section" with an aspect ratio of  $1 \div 1.62$  ( $31 \div 50$  mm) and a rectangle with elongated proportions of  $1 \div 2.31$  ( $26 \div 60$  mm).



*Fig. 13. The study of the perception of the "golden section" on the example of rectangles of different proportions*

When choosing rectangles in the normal state, a square is preferred 1/2 the time. The right hemisphere prefers the golden ratio and rejects the elongated rectangle. Conversely, the left hemisphere gravitates toward elongated proportions and rejects the golden ratio.

During copying of these rectangles the following was observed: when the right hemisphere was active, the proportions in the copies were most exact; when the left hemisphere was active, the proportions of all rectangles were distorted, the rectangles were stretched (the square was drawn as a rectangle with a side ratio of  $1 \div 1.2$ ; the stretched rectangle proportions increased dramatically and reached  $1 \div 2.8$ ). The proportions of the "golden" rectangle that were most distorted; when proportions in the copies became a rectangle  $1 \div 2.08$ .

When making their own drawings, proportions close to the golden ratio and elongated proportions prevail. On average, the proportions are  $1 \div 2$ , with the right hemisphere favoring the golden section proportions and the left hemisphere departing from the golden section proportions and elongating the drawing.

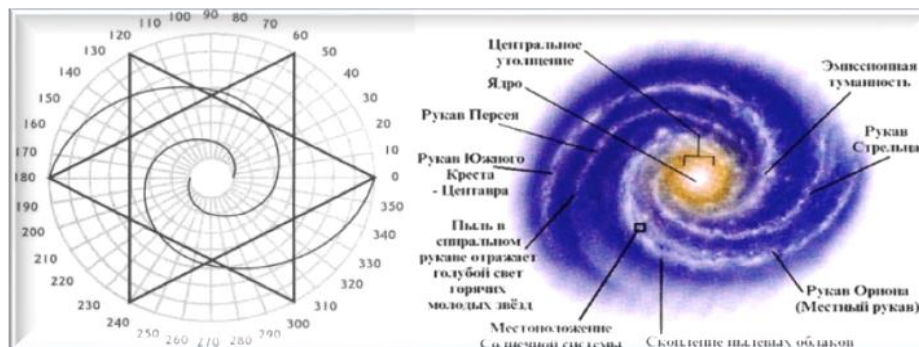
Now draw some rectangles, measure their sides and find the aspect ratio. Which hemisphere is your dominant hemisphere?

### The golden ratio and the cosmos

It is known from the history of astronomy that J. Titius, a German astronomer of the 18th century, used this series to find a regularity and order in the distances between the planets of the solar system.

However, there was one case that seemed to contradict the law: there was no planet between Mars and Jupiter. Focused observation of this section of the sky led to the discovery of the asteroid belt. This happened after the death of J. Titius in the early 19th century. The Fibonacci sequence is widely used: it is used to represent the architectonics of living beings, man-made structures, and the structure of galaxies. These facts are the evidence of independence of the number series from the conditions of its manifestation, which is one of the signs of its universality.

Two Galactic Golden Spirals are compatible with the Star of David (Fig. 14).



*Fig. 14. A galaxy in the form of a spiral corresponding to the formula of the golden section*

Pay attention to the stars coming out of the galaxy in a white spiral. It is exactly 1,800 meters from one of the spirals, until another unfolding spiral comes out... For a long time, astronomers simply believed that everything that everything we see is all that is there; if something is visible, then it exists. They either did not notice the invisible part of Reality at all, or they did not consider it important. But the invisible side of our Reality is actually much larger than the visible side and probably more important... In other words, the visible part of Reality is much less than one percent of the whole - almost nothing. In fact, our real home is the invisible universe...

In the Universe, all the galaxies known to mankind and all the bodies in them exist in the form of a spiral corresponding to the formula of the golden section. In the spiral of our galaxy lies the coefficient of the golden section.

### **Conclusion**

Nature, understood as the whole world in the diversity of its forms, consists, as it were, of two parts: living and inanimate nature. The creations of inanimate nature are characterized by high stability, weak variability, if judged on the scale of human life. A person is born, lives, grows old, dies, but the granite mountains remain the same and the planets revolve around the Sun in the same way as in the time of Pythagoras.

The world of living nature appears to us completely different - mobile, changeable and surprisingly diverse. Life shows us a fantastic carnival of diversity and uniqueness of creative combinations! The world of inanimate nature is primarily a world of symmetry, which gives stability and beauty to its creations. The world of nature is primarily a world of harmony, in which the "law of the golden section" operates.

In the modern world, science is gaining special importance, due to the increased human impact on nature. Important tasks at the present stage are the search for new ways of coexistence between man and nature, the study of the philosophical, social, economic, educational and other problems facing society.

In this paper, the influence of the properties of the "golden section" on living and non-living nature, on the historical course of the development of the history of mankind and the planet as a whole was considered. Analyzing all of the above, one can once again marvel at the grandiosity of the process of cognition of the world, the discovery of all its new laws and draw a conclusion:

The golden ratio principle is the highest manifestation of the structural and functional perfection of the whole and its parts in art, science, technology and nature.



It can be expected that the laws of development of various systems of nature and the laws of growth are not very diverse and can be traced in a variety of formations. This is where the unity of nature manifests itself. The idea of such unity, based on the manifestation of the same laws in heterogeneous phenomena of nature, has remained relevant from Pythagoras to the present day.

**October 2019**

**F. D. Shkrudnev**